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ON TWO - DIMENSIONAL
ARMA MODELS
FOR IMAGE ANALYSIS

by

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ON TWO-DIMENSIONAL ARMA MODELS
FOR IMAGE ANALYSIS

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ABSTRACT

The two-dimensional autoregressive moving-average (2-D ARMA) models
have been useful for image coding and compression, statistical image modeling,
image feature extraction and segmentation, texture characterization, and image
restoration and enhancement. This paper provides a critical review of the
2-D ARMA models for image analysis. Particular emphasis is placed on restora-
tion of noisy images using 2-D ARMA models. Computer results are presented
for comparative evaluation study. Problem areas such as order determination
are examined. Although there are shortcomings and unsolved problems it is
concluded that the models are very effective linear models for image analysis.

I. Introduction

The 2-D ARMA models are now playing an increasingly important role

in image analysis just as the linear prediction is important in speech

signal processing. The primary advantage of the models is the good linear

approximation to real data. For one-dimensional data, the models are now

well developed. Extension of the one-dimensional to two-dimensional models

is not straightforward however. In fact much work remains to be done on

2-D ARMA models for image analysis. For examples, efficient procedures to

determine the coefficients (or parameters) of the two-dimensional autoreg-

ressive (AR), moving average, ARMA processes are much needed. The window

size or the order determination is another problem which does not yet have

a good solution. The models are very suitable for homogeneous random field.

For most images in practical applications, certain objects are to be extracted

from background. The models are not suitable for object boundary representation. The simplicity of linear model however outweighs the shortcomings and problems mentioned above. The bibliography at the end of this paper provides a list of references on recent work in this area.

II. Applications of 2-D ARMA Models to Image Analysis

Let X_{rs} be the gray level of the picture element at rth row and sth column of a digitized picture. For the nth order autoregressive model, the picture elements up to a distance of n should be included in the model. The general expression for non-causal models is given by:

(1)

$$X_{rs} = \sum_{i=1}^{n} a_i (x_{r-1,s} + x_{r+1,s}) + \sum_{i=1}^{n} b_i (x_{r,s-1} + x_{r,s+1}) + w_{rs}$$

where w_{rs} is a zero mean input Gaussian sequence with finite variance which may or may not be known. For causal models, only past history terms should be included in Eq. (1). The parameters a_i and b_i can be estimated by using the principle of minimum mean square error estimation if the causal model is considered. For the ARMA models, w_{rs} should be replaced by a set of terms corresponding to the past input sequence.

In image coding and data compression [8,9,11], the estimated parameters values of a_i and b_i along with the initial conditions can be used for image transmission. This represents a very significant data reduction over the use of original image data. The parameters and initial conditions can be coded with established coding schemes. Good quality regenerated picture is available from a second order AR model for Guass-Markov field [9].

In statistical image modeling, ARMA models take into account the contextural information, from nearest neighbors [1,2,3,5,12,16,21]. In the first order AR model, for example, one of the parameters is for the inter-pixel correlation. Even though the images are rich in contextual information, most dependence is from neighboring pixels (picture elements). Thus low order ARMA models can still be effective to represent the image. The limitation to the study of causal or semi-causal models is a drawback as each pixel should depend on its neighbors in all directions. For modeling of an object-plus-background scene, an adaptive procedure is needed so that the estimated parameters will be adjusted near object boundaries.

In image feature extraction and classification, the parameters of the ARMA models are not very suitable as features for image classification because the joint probability density of the parameters is unknown. A conditional probability density of the gray levels of picture elements under consideration can be determined from estimated parameters and the known distribution of w_{rs} [16]. However, the number of pixels considered is necessarily large and it is not convenient to work with a very high dimensional space. For example, a dimension of 25 is large for a small 5 x 5 subimage.

In image segmentation, subimages of similar statistical characteristics tend to have similar ARMA models. The models can be useful to determine region merging in the segmentation process. In model building for pictorial textures, appropriate ARMA model can be identified [20] by least square estimation by minimizing the sum of squared residuals. For texture synthesis, a noise generator is used to generate an array of noise from which a new array

of data x_{rs} is computed according to the statistical model. The ARMA models appear to be particularly suitable for pictorial textures study [7,14,19,20].

In restoration or enhancement from noisy images, the estimated parameters of the ARMA are useful to reconstruct the original image and thus performing effective filtering operations [6,10,13,15,17,18]. Significant improvement in signal-to-noise ratio is available even for the first order models [6].

III. On a 2-D ARMA Model for Noise Filtering

The model considered here is based on that of Katayama [13]. Assume a 2-D homogeneous image with the autocovariance function,

$$R_{xx}(i,j) = \sigma_x^2 \exp \{-C_1|i| - C_2 |j|\}; C_1, C_2 > 0 \qquad (2)$$

The image field with this autocovariance function can be modeled by

$$x_{r+1,s+1} = a_1 x_{r,s+1} + a_2 x_{r+1,s} - a_1 a_2 x_{r,s} + w_{r,s} \qquad (3)$$

where $w_{r,s}$ is a Gaussian white noise field with zero mean and variance q.

$$a_1 = \exp(-C_1), a_2 = \exp(-C_2), q = \sigma_x^2(1-a_1^2)(1-a_2^2)$$

Now consider the observable noisy image,

$$Y_{r,s} = X_{r,s} + v_{r,s} \qquad (r = 1,2,---,M; s = 1,2,---,N) \qquad (4)$$

where $v_{r,s}$ is a Gaussian white noise field with mean zero and variance σ_v^2. The model of (3) and (4) forms a state-space model for the noisy images. For the purpose of the parameter identification, form a 2-D ARMA model for the noisy image,

$$Y_{r+1,s+1} = a_1 Y_{r,s+1} + a_2 Y_{r+1,s} - a_1 a_2 Y_{r,s} + v_{r+1,s+1}$$

$$+ (K-1) [a_1 v_{r,s+1} + a_2 v_{r+1,s} - a_1 a_2 v_{r,s}]$$

where K is a stationary gain, $0 < a_1, a_2, K < 1$. Let $\theta = (a_1, a_2, K)$.
Define the sample variance of $v_{r,s}$ as

$$\Lambda(\theta) = \frac{1}{MN} \sum_{r=1}^{M} \sum_{s=1}^{N} v_{r,s}^2$$

We identify the parameters a_1, a_2, and K such that $\Lambda(\theta)$ is minimized.
The method of Steepest Decent is used in the optimization process which
is far more efficient computationally than the optimization procedure
employed in [13].

Figures 1 & 2 are two subimages (150 x 150) considered along with
their histograms. Figure 3 is a noisy image corresponding to Figure 1
with SNR = 1.73. Six iterations are performed in each image in the optimi-
zation process. Figures 4 & 5 are the results of filtering for different
initial conditions. The procedure converges very well for any initial
conditions. Figure 6 has the SNR = 0.87 with filtered result given by
Figure 7. Figure 8 is the noisy image corresponding to Figure 2 with
SNR = 1.73. Figure 9 is the filtered result. Figure 10 has SNR = 0.87
with filtered result given by Figure 11. All filtered results accompanied
by the histograms of the filtered images clearly illustrate that the 2-D
ARMA model provides a very effective filtering procedure for the noisy
images.

The Kalman filtering performed line-by-line is much faster than the
use of ARMA model while providing good filtered result. Figure 12 is an
original image (300 x 400) considered along with its histogram. Noise

is added to Figure 12 so that the SNR ≈ 1.73. Figure 13 is the result of Kalaman filtering of the noisy image performed horizontally line-by-line [6]. The filtered image is nearly identical to the original image without noise. Kalman filtering of the 300 x 400 image requires only one-fourth of the time required for the model. It is concluded that the Kalman filtering is superior to the ARMA model for restoration of noisy images.

IV. The Order Determination Problem

Although, the Akaike information criterion can be used to determine the window size or model order in horizontal and vertical directions separately [5]. Maximum likelihood decision rule has been suggested for choice of neighbors. For the subimage of Figure 1, we have employed the procedure developed by Penm and Kanefsky [15] to determine the order accurately. The idea is to examine the determinant of the matrix of sample correlation functions defined by

$$C_{r,s} = \frac{1}{(M - r)(N - s)} \sum_{i=r+1}^{M} \sum_{j=s+1}^{N} x_{i,j} \, x_{i-r,j-s} \tag{5}$$

where $X_{i,j}$ is an element of the data in a window of size M x N. The values of M and N are increased until the determinant falls below certain specified value. The largest M and N values correspond to the window sizes in horizontal and vertical directions respectively. The result of the computation shows that the order should be one in each direction. Thus the ARMA model should have an order of 2 which shows that the assumption of the second order model in the previous section is indeed valid.

V. Concluding Remarks

In this paper we have examined critically the feasiblity of 2-D ARMA models for various image analysis studies. On the whole the models are

very effective though they do not necessarily perform the best. More research is much needed on the 2-D ARMA models so that the models can serve as a general purpose image analysis tool which is fairly independent of the specific image application under consideration.

BIBLIOGRAPHY

This bibliography includes only the recent work on the use of 2-D ARMA models in image analysis.

1. N. Ahuja and A. Rosenfeld, "Image models," TR-781, Computer Vision Lab, University of Maryland, July, 1979.

2. M. S. Bartlett, "The Statistical Analysis of Spatial Patterns," Chapman and Hall, London and Wiley, New York, 1975.

3. R. Chellappa and N. Ahuja, "Statistical inference theory applied to image modeling," TR-745, Computer Science Center, University of Maryland, March, 1979.

4. R. Chellappa, R. L. Kashyap and N. Ahuja, "Decision rules for choice of neighbors in random field models of images," Computer Science Technical Report Series TR-802, University of Maryland, August, 1979.

5. C. H. Chen, "Statistical image modeling," TR-EE-6, Southeastern Massachusetts University, October, 1979. Also presented at the ENAR Joint Statistical Meetings, Charleston, S.C., March 1980.

6. C. H. Chen and J. Chen, "A comparison of image enhancement techniques," TR-EE-8, Southeastern Massachusetts University, December, 1979.

7. K. Deguchi and I. Morishita, "Texture characterization and texture-based image partitioning using two-dimensional linear estimation techniques," IEEE Trans. on Computers, vol. C-27, August, 1978.

8. K. Duguchi and I. Morishita, "Image coding and reconstruction by two - dimensional optimal linear estimation," Proc. of the Fourth International Joint Conference on Pattern Recognition, pp. 530-532, November, 1978.

9. E. J. Delp, R. L. Kashyap and O. R. Mitchell, "Image data compression using autoregressive time series models," Pattern Recognition, vol. 11, no. 5/6, pp. 313-323, 1979.

10. A. Habibi, "Two-dimensional Bayesian estimate of images," Proc. of the IEEE, vol. 60, no. 7, pp. 878-884, July, 1972.

11. A. K. Jain, "Image coding via a nearest neighbors image model," IEEE Trans. on Communications, vol. COM-23, pp. 318-331, March 1976.

12. R. L. Kashyap, "Univariate and multi-variate random field models for images," Workshop on Image Modeling, Chicago, August, 1979.

13. T. Katayama, "Restoration of noisy images using a two-dimensional linear model," IEEE Trans. on Systems, Man, and Cybernetics, vol. SMC-9, no. 11, pp. 711-717, November, 1979.

14. B. H. McCormick and S. N. Jayarmamurthy, "Time series model for texture synthesis," International Journal of Computer and Information Sciences, vol. 3, pp. 329-343, 1974.

15. H. W. Penm and M. Kanefsky, "Identifying a two-dimensional ARMA model for chromosome pictures," presented at the Workshop on Pattern Recognition and Artificial Intelligence, Princeton, N.J., April, 1978.

16. S. L. Sclove, "Pattern recognition in image processing using inter-pixel correlation," Report prepared for Grant AFOSR 77-3454, Department of Mathematics, University of Illinois at Chicago Circle, Augues, 1978.

17. M. G. Strintzis, "Dynamic modeling of cyclic processes with application to two-dimensional image restoration," Proc. of Conference on Information Science and Systems, Johns Hopkins University, pp. 88-90, 1976.

18. M. G. Strintzis, "Comments on two-dimensional Bayesian estimate of images," Proc. of IEEE, vol. 64, pp. 1255-1256, August, 1976.

19. J. T. Tou and Y. S. Chang, "An approach to texture pattern analysis and recognition," Proc. of IEEE Conference on Decision and Control, pp. 398-403, 1976.

20. J. T. Tou, D. B. Kao, and Y. S. Chang, "Pictorial texture analysis and synthesis," Proceedings of the Third International Joint Conference on Pattern Recognition, p. 590, November, 1976.

21. P. Whittle, "On stationary processes in the plane," Biometrika, vol. 41, pp. 434-449, 1954.
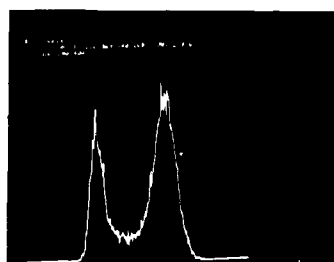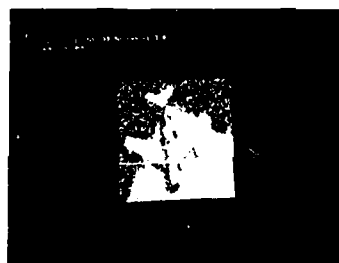
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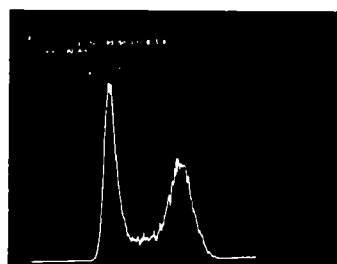
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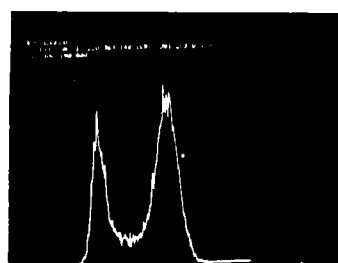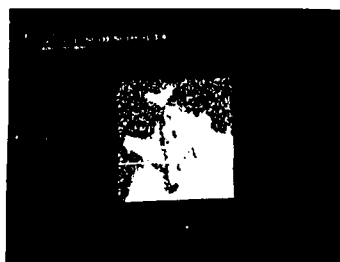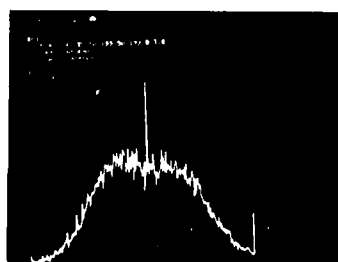
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Fig. 3a

Fig. 3b

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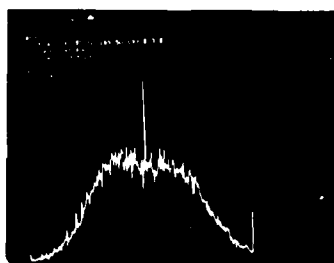
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Fig. 5a

Fig. 5b

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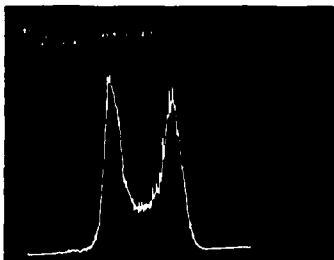
Fig. 1a

Fig. 1b

Fig. 2a

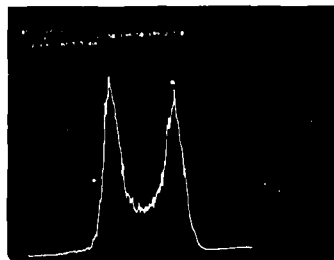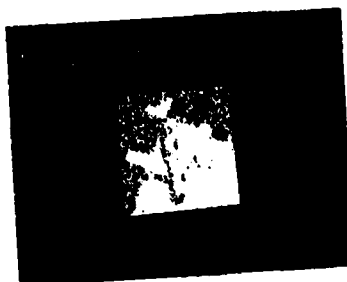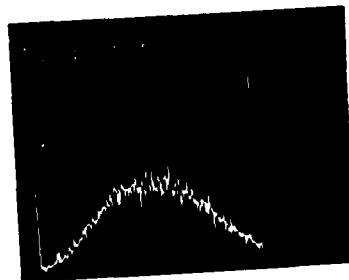Fig. 2b

Fig. 3a

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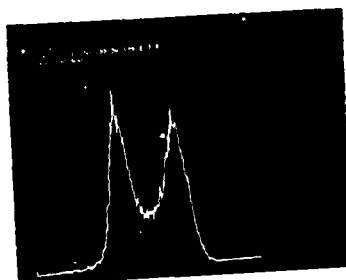
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Fig. 5a

Fig. 5b
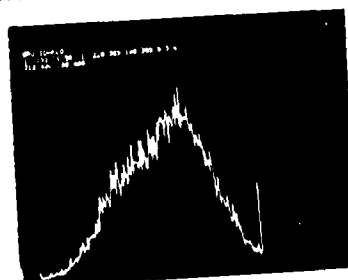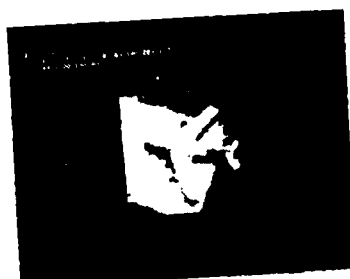
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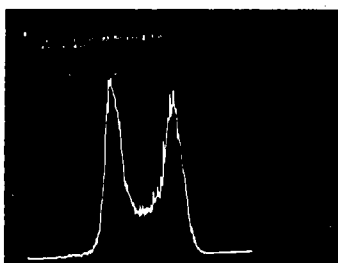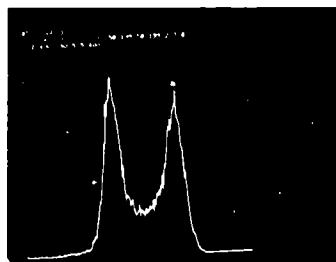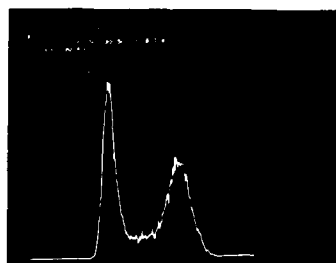
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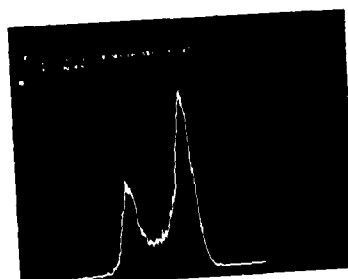
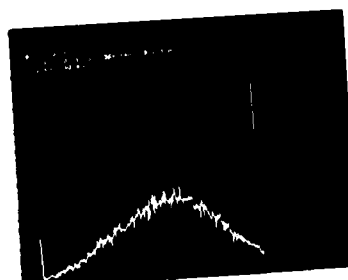Fig. 7a

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Fig. 8a

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Fig. 10a

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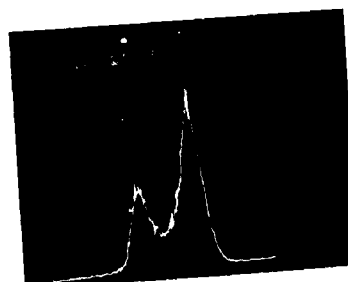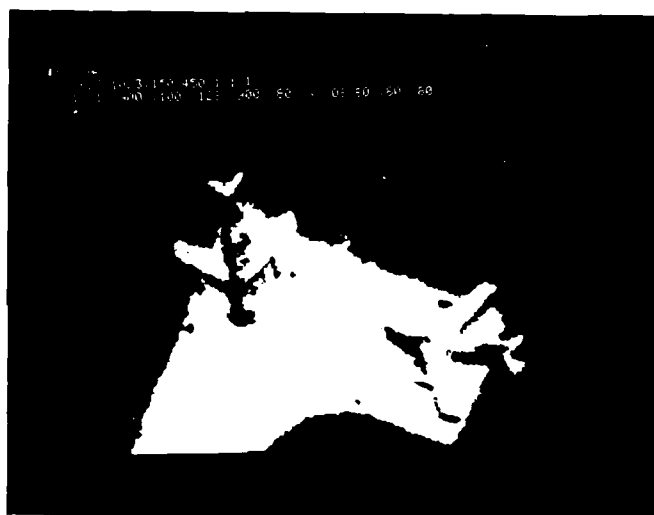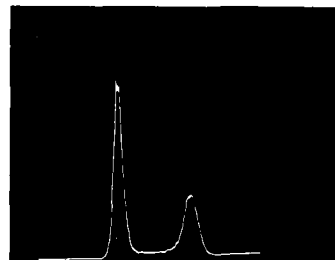
Fig. 11a

Fig. 11b

F2

Fig. 12a

Fig. 12b

Fig. 13a
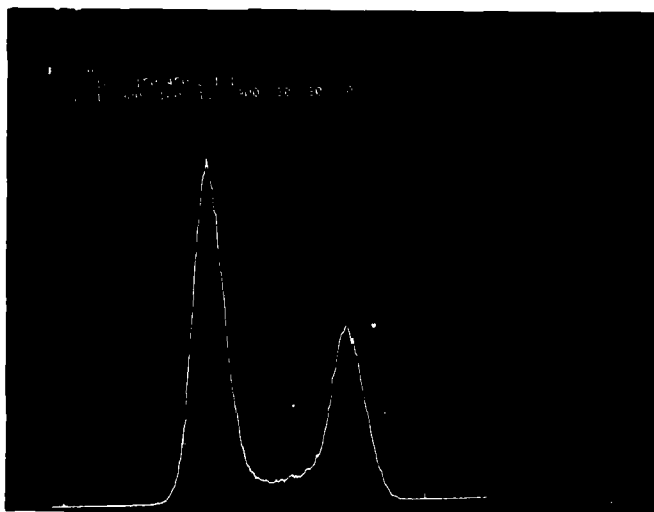
Fig. 13b

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
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